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AIR MINISTRY
METEOROLOGICAL RESEARCH COMMITTEE

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The representation of the annual temperature variation over Central and Northern Europe by a two term harmonic form

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Summary

A harmonic analysis is made of the annual temperature variation at 42 stations spread over Central and Northern Europe, using in each case 5-day mean temperature averaged over a considerable number of years. It is found that the first two harmonics account for nearly all the annual variance, the residual variance being usually less than 0.4 per cent of the total. Most of this residual variance seems to be random in origin, but a small part may be due to systematic departures from the two term harmonic form.

In about 10 of the 73 5-day periods which cover the year, it is found that departures from the two term harmonic form are in the same direction at nearly all the stations considered, and similar departures are found in the same periods in data for independent periods of years.

1. Introduction

The present paper covers some of the groundwork of Item II.5(b) of the Meteorological Research Programme "Studies of the variations of weather conditions over the British Isles with "periods" of up to about one month, and their relationships with the large scale circulation". A natural approach to this problem is to choose as many cases as possible in which some weather element takes an extreme value, for example, periods of unusually cold weather in the British Isles during spring and find as much as possible about them; how far does the cold weather extend, and how long does it last, and is it accompanied or preceded by abnormalities in the values of any other weather elements, especially in any element indicative of the state of the general circulation? To answer these questions we must know the normal values of the weather elements, not only for the British Isles, but for an area covering at least a large sector of the Northern Hemisphere, and for unit periods, not of calendar months, but of much shorter periods of only a few days. Finally, when we have prepared our anomaly charts, we may find, for example, that cold periods in spring in London are usually preceded by unusually high pressure in Greenland. Out of the myriad of such possible relations we have to select those which are statistically significant, and may be expected to recur in the future. The process of separating the significant relationships from the casual is a necessary and inescapable part of the research, and to do it we have to have representative normal values of the weather elements over the whole vast area which concerns us. For most of the area the only basic observations we have to go on are daily synoptic observations plotted on charts, and tabulated monthly mean values, and neither source will yield directly what we want, namely, mean values and standard deviations, (or other suitable measures of variability) for periods of less than one month for a network of stations covering a large part of the Northern Hemisphere. The weather elements which interest us include

/surface

surface pressure, surface temperature, rainfall, sunshine and upper air temperature and we have to estimate the normal value of any of these for periods of only a few days. Such estimates may be made subjectively, based on observed monthly means, but this introduces an avoidable source of error. Instead we propose to use harmonic analysis to find the normal form of the annual variation of each meteorological element, and hence the normal value for any day or period of days.

The problem can be dealt with in three stages, namely

(1) an investigation of daily values of the element concerned, to find how many harmonic terms must be taken to obtain an acceptable fit,

(2) a harmonic analysis to find the required number of terms from observed monthly mean values for each station we wish to use, and

(3) a harmonic synthesis to obtain the normal value for any period which interests us.

The present report covers the first stage of the investigation of the first element studied, namely, mean daily air temperature. The second and third stages will be covered in a later paper. The processes used can be applied to other weather elements with only slight modifications.

We propose to fit the annual temperature variation by an expression of the form

$$T = a_0 + \sum_{j=1}^{N} (a_j \cos j\theta + b_j \sin j\theta) \qquad \text{----------} \quad (1)$$

where θ increases by 2π during the year, and in this work is measured from zero at midnight on December 31st. (The year is regarded as having exactly 365 days, February 29th being ignored in leap years.) An expression of the form given can, of course, be made to fit any series of observed data by taking N large enough, but we have no justification for seeking an exact fit to any series obtained by averaging observed temperatures over a finite number of years. The mean of the temperatures observed during a finite number of years will differ from the hypothetical normal by a sampling error depending on the variation between one year and another, and the normal form of the annual temperature variation will be given as accurately as the data will allow by any function which does not differ from the observed averages at any time of year by amounts which singly or collectively exceed the limits fixed by the hazards of sampling.

If we do fit exactly a series of observations covering a year, and V(T) denotes the variance of temperature during the year, then a standard theorem states that

$$V(T) = \tfrac{1}{2}(a_1^2 + b_1^2 + a_2^2 + b_2^2 + \ldots\ldots) \qquad \text{----------} \quad (2)$$

Thus we can, by calculating expressions such as $V(T) - \tfrac{1}{2}(a_1^2 + b_1^2)$, decide how much of the total variance is due to the first harmonic, or due to whatever combination of harmonics we have chosen to calculate, and since these expressions are essentially positive, we can fix an upper limit on the amplitude of any harmonic we have not calculated so far. This relation is useful in showing us where to stop.

/2.

2. Data and method

For completeness, the investigation should start with daily mean temperatures for every day of the year. However, an investigation which started on these lines would be handicapped by the labour involved, and by the difficulty of finding enough suitable data. There are, however, many publications containing mean temperatures for the 73 non-overlapping 5-day periods which cover the year, and it is unlikely that any temperature anomaly which is smoothed out by averaging over a period as short as five days will recur on the same day from year to year. So the present investigation was based on 5-day mean temperatures, in the expectation that any significant and annually recurring feature of the sequence of daily mean temperatures would also be present in the 5-day means. Table 1 gives particulars of the stations used, and the sources from which the data were obtained.

3. The calculation

The calculation of harmonic coefficients from 73 equally spaced observations is not difficult, but it is laborious and hard to check. The formulae are,

$$a_k = 2/73 \sum_{j=1}^{73} T_j \cos \theta_j \qquad \text{---------- (3)}$$

and

$$b_k = 2/73 \sum_{j=1}^{73} T_j \sin \theta_j \qquad \text{---------- (4)}$$

where k=0,1 or 2 and T_j, θ_j are the observed mean temperature and the angle corresponding to the j-th 5-day period of the year. Here θ_1 is the angle corresponding to the middle of the first 5-day period, Jan. 1st to 5th. The values of $\cos \theta_j$, $\sin \theta_j$ are given by Pollak (1948) and these were multiplied by the factor 2/73 and incorporated in a special computing form prepared in M.O.22, of which a specimen is shown in Fig.1(a). The 5-day mean temperatures are written into another form shown in Fig.1(b) which also contains spaces for all numbers required in calculation. Corresponding columns on the two forms are brought together and the products are formed and summed on a desk machine. Experience shows that arranged in this way the calculation is reasonably easy to check. For about 8 stations the harmonic analysis was carried out in this way: later, data for 34 more stations were analysed by punched card methods, taking advantage of facilities for processing the cards offered by Rothamsted Experimental Station. In Table 2 are given the coefficients of the first two harmonics for each set of data, the total annual variance, and the residual variance after subtracting that due to the first two harmonics.

4. The geographical variation of the harmonic parameters

The semi amplitude and phase of the first and second harmonics are shown in Figs.2 (a) to 2(d). Besides the parameters calculated in the present work values have been obtained from two other sources. The parameters for 18 Norwegian stations are given by Birkeland (1936) while those for 160 stations in the British Isles have been calculated in M.O.22. These values which are computed from monthly mean temperatures are somewhat less reliable than those computed from 5-day mean temperatures. In Fig.2(e) are given the names and positions of the original stations and those used by Birkeland, together with their altitudes and mean annual temperatures. Fuller information about the stations used by Birkeland is given in Tables 1(a) and 2(a).

/Fig.2(a)

Fig.2(a) shows the semi amplitude of the first harmonic. The amplitude increases generally towards the east, but there is a striking difference between the rapid increase as the Norwegian mountains are crossed compared with the much slower increase across the North German plain. In Fig.2(b) the phase of the first harmonic is given by the calendar date on which the maximum is attained. The German data show with surprising consistency that the phase is earlier in the interior of the Continent than in coastal regions. This association is borne out by the dense network of stations over the British Isles, which shows, moreover, that the change of phase is most rapid in a belt of perhaps 20 or 30 miles inland from the coasts. The four German stations which are bracketed are mountain stations, and at these the amplitude is smaller and the phase later than at nearby stations at lower altitudes. In these respects the mountain stations seem to have a more maritime climate than their neighbours. Figs.2(c) and 2(d) give the semi-amplitude and phase of the second harmonic. The data are self consistent enough to suggest that the variations shown are not due to chance, but we can hardly from these data alone suggest what the cause or large scale character of the variation may be. The variations of the harmonic parameters with geographical situation and with altitude deserve further study, using more stations spread over a larger area.

5. The magnitude of the residual variance

The residual variance is given in Table 2 both in absolute units ($^{o}C^{2}$) and expressed as a percentage of the total annual variance. It seems to be disproportionately large at three of the four mountain stations, and if we exclude these its value is never greater than 0.23 $^{o}C^{2}$, and in some places is as low as 0.05 $^{o}C^{2}$. Expressed as a percentage, the residual variance, except at the mountain stations, is generally less than 0.4 per cent. These figures give an upper limit to the order of magnitude of the errors we accept if we assume that the annual march of temperature is given by the best fitting two term harmonic form.* For some purposes these errors may be negligible, for example, in long range forecasting, the errors in predicting the future value of the temperature anomaly on a given day will probably far exceed the errors in predicting the normal temperature for that day. At any rate the investigator, knowing their probable magnitude, can decide whether to ignore them or not.

These results complete the main purpose of the paper, which is to give some idea what to expect if the annual temperature variation is assumed to follow the two term harmonic form. The rest of the paper sets out to find a little more about the way in which the residual variance is made up. The data are not very suitable for this purpose, and the work does not claim to be exhaustive, but simply to examine some of the more obvious indications which may be extended and confirmed by later more extensive and laborious investigation.

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* The corresponding values in Fig.2(a) give for Birkeland's Norwegian data the corresponding errors involved in omitting the third to sixth harmonics - they are mostly lower than our estimates of the total residual variance, as is to be expected.

6. The dependence of the residual variance on the place and the period over which the data are averaged

Suppose we make a two term harmonic analysis of the 5-day mean temperatures for one station for each of a number of years, and then do the same for the same data averaged over, say, 30 years. The harmonic coefficients for the 30-year mean data would be the mean of those for the individual years, and while they will show some sampling variation, depending on which 30 years are chosen, this variation will probably be small. In any case, we cannot investigate it with our present data, which consist only of the mean values for long periods of years, and not the temperatures for 5-day periods of the individual years. The residual variance for each single year may be made up of two parts, A which measures the departure of the individual year from the true normal, and B, which measures the departure of the true normal from the best fitting two term harmonic form. Experience suggests that in an individual year A is far larger than the value found in Section 5 from the mean of many years, and also that the departures in one year do not bear any easily recognisable relation to those in any other year. It is a reasonable hypothesis that A is a random variable. If so, and if we write V(n) for the residual variance for data averaged over n years, then

$$V(n) = A/n + B \qquad (5)$$

or

$$nV(n) = A + Bn \qquad (6).$$

These equations suggest that V(n) should tend to decrease as n increases (and the values in Table 2 suggest this), while nV(n) should either be constant or should tend to increase with n. Except for Halle and Prague, we have only one value of nV(n) for each station, and these are plotted against n in Fig.3, while their geographical distribution is shown in Fig.4. The mountain stations (ringed) do not agree with the rest and if these are omitted, then Fig.3 does suggest some slight increase of nV(n) with n. The regression line giving the best estimate of nV(n) in terms of n is

$$nV(n) = 12.1 + 0.14n \qquad (7)$$

but the coefficient of n on the right hand side is not significantly different from zero. Turning now to Fig.4, there is a strong suggestion of a geographical variation of nV(n) with a maximum in central and north-western Germany. Most of the 80-year mean values on Fig.3 come from German stations, so this geographical variation accounts for some and perhaps all of the apparent increase of nV(n) with n. To sum up, the data are consistent with the residual variance in each individual year being an entirely random quantity, but they are also consistent with the existence of slight systematic departures of the true form of the annual temperature variation from the two term harmonic form. If such systematic departures exist, their contribution to the total annual variance averaged over these stations is probably less than 0.14 per cent. If in Equation (7) we put n equal to 50 or less, the first (random) component of the residual variance will outweigh the second, so that when we are estimating temperature normals from records covering less than 50 years, and in particular, if we are using data for the internationally agreed period of 30 years, the two term harmonic form will give as accurate an estimate of the normal temperature as the data themselves will justify.

7. Departures from the two term harmonic form at particular times of year

Even if departures from the two term harmonic form are not important on the average in estimating the normal temperature they may be important at particular times of year, and they are of interest in themselves, even if very small, provided they can be shown to be systematic and permanent, since they may focus attention on periods of the year when synoptic situations peculiar to the season are to be expected.

/The

The nature of these departures is shown in Fig.5 which shows superimposed curves of the observed mean temperature and the best fitting two term harmonic form for eight stations spread over the area covered by the observations. The horizontal scale, which is the same for all stations, gives the time, in 5-day units, numbered from the beginning of the year, while the vertical scales, which are in degrees Celsius, have been separated to avoid confusion. Thus anomalies for different stations for the same period appear in the same vertical line. It appears that the anomalies for any station for consecutive 5-day periods are serially correlated: in cases worked out later (Table 6) the first serial correlations are mostly between 0.5 and 0.7. There is also a suggestion of spatial correlation: that is, the anomaly at a particular period of the year tends to have the same sign over a large part or the whole of the area for which we have data. These temporal and spatial correlations are what we would expect if the anomalies are due, not to fortuitous or local effects, but to large scale features of the atmospheric circulation, and they must be allowed for when we decide on the statistical significance of the anomalies. For example, a single exceptional winter may produce a substantial negative contribution to the temperature anomalies for stations over a large area for several consecutive periods, and if we treat all these anomalies as independent we will arrive at a wholly unjustified degree of confidence that similar anomalies will repeat themselves in future. However, to start with, we will examine the eight records to see what they suggest, without at this stage trying to show whether these suggestions are significant. The features which deserve further investigation are given in Table 3.

Table 3. 5-day periods in which the temperature anomalies are remarkably consistent in sign at the 8 stations

Number of period	Direction of anomaly	Where present
No. 1	Warm	All except Tilsit
No. 2	Warm	All except Glasgow
No. 7	Warm	All 8 stations
No. 9	Cold	All except Kew and Glasgow
No. 15	Cold	All 8 stations
No. 19	Warm	All 8 stations
No. 24	Cold	All except Uppsala and Tilsit
No. 29	Warm	All except Vienna and Glasgow
No. 30	Warm	All 8 stations
No. 31	Warm	All except Uppsala
No. 33	Cold	All except Tilsit
Nos. 34	Cold	All except Glasgow
35	Cold	All 8 stations
No. 55	Warm	All 8 stations
No. 60	Cold	All except Tilsit and Uppsala
No. 62	Warm	All except Potsdam
No. 64	Cold	All except Uppsala
No. 65	Cold	All 8 stations
No. 73	Warm	All except Glasgow and Uppsala

If the anomalies at the eight stations were unrelated, then the expectation that all eight for a given period should agree in sign is 1/128, and the

/expectation

expectation that seven of the eight should agree is 1/16. Thus when we examine 73 periods we would expect to find perhaps one period in which all eight agree, and four or five periods in which seven of the eight agree. In fact there are seven periods in which all eight agree, and seven others in which seven of the eight agree, so there is a presumption that the agreements are not wholly due to chance. This presumption is increased when we note, from Table 3, that the exceptional stations, when seven of the eight agree, are nearly always stations on the edge of the area considered, namely, Glasgow, Uppsala or Tilsit. This feature is explicable if the weather processes which produce the anomalies are on a scale usually not quite big enough to affect the whole of this area in the same way.

If it is accepted that during the period to which most of these data relate (the 80 years 1851 to 1930) there are significant departures over this area from the two term harmonic form, we yet have to consider whether these are likely to be parmanent, or to arise from the peculiarities of that particular period. The features likely to be permanent at a place should be identifiable by comparing two records for that place for independent periods of years. Such records are not easy to find, but two have been found, namely, for Prague (Fig.6) and for Halle (Fig.7), while there are also mainly non-overlapping records for two pairs of stations which are sufficiently close for us to presume that they will be affected alike in any large scale process. These pairs are Edinburgh and Glasgow (Fig.8) and Brussels and Kleve (Fig.9).

When we compare the anomalies for the periods listed in Table 3 with those in Figs.6 to 9, we find that in most cases the anomalies are repeated in the records for independent periods of years. In particular it is hardly possible to doubt that for the periods listed in Table 4 the anomalies are systematic and permanent departures from the two term harmonic form.

Table 4

Number of 5-day period	Dates	Direction of anomaly	Remarks
No. 7	Jan. 31 - Feb. 4	Warm	Present in every record, though very small in one or two.
No.15	Mar. 12-16	Cold	Present in all except Edinburgh 1795-1850
No.19	Apr. 1-5	Warm	Present in all except Prague 1775-1839
No.29	May 21-25	Warm	Present in all except Vienna and Glasgow, where it is zero
No.30	May 26-30	Warm	Present at all stations
No.31	May 31 - June 4	Warm	Present at all stations except Prague and Uppsala
No.35	June 20-24	Cold	Present at all except Brussels 1831-1860
No.55	Sept.28 - Oct.2	Warm	Present in all stations
No.64	Nov.12-16	Cold	Present at all except Uppsala
No.65	Nov.17-21	Cold	Present at all stations

One other deduction can be made from Figs.6 to 9. If these anomalies are partly non-random in origin, then there should be a positive correlation between the anomalies found from comparable records for non-overlapping periods of years. These correlations are given in Table 5.

/Table 5.

Table 5. Correlations between anomalies for wholly or mainly independent periods of years

First station	Second station	Correlation
Glasgow 1868-1912	Edinburgh 1795-1804 and 1821-1850	0.05
Brussels 1831-1860	Kleve 1851-1930	0.35
Prague 1775-1839	Prague 1840-1934	0.29
Halle 1851-1900	Halle 1901-1950	0.29

The correlation between the anomalies at Edinburgh and Glasgow is evidently not significant: the significance of the other three must be judged in the light of the auto-correlations which exist in the various series of anomalies, the first serial correlations of which are given in Table 6.

Table 6. Auto-correlations of multi year averages of 5-day mean temperatures

Station	Period	Correlation
Glasgow	1868 - 1912	0.15
Edinburgh	1795-1804 and 1821 - 1850	0.46
Kleve	1851 - 1930	0.56
Brussels	1831 - 1860	0.49
Prague	1775 - 1839	0.72
Prague	1840 - 1934	0.56
Halle	1851 - 1900	0.45
Halle	1901 - 1950	0.47

Clearly we cannot treat the correlations given in Table 5 as derived from 73 independent pairs of observations. An estimate following the lines of work by Lewis and McIntosh (1952) gives the "equivalent number of repetitions" as between 3 and 5, so that the correlations in Table 5 may be judged as based on from 15 to 25 independent pairs of observations. On this basis none of them is significant even at the 10 per cent level. However, the three values for the continental stations are large enough to be suggestive, particularly as the periods of the three comparisons are virtually independent. The absence of association between the anomalies for Edinburgh and Glasgow agrees with the conclusion of Brooks and Mirrlees (1930) who made a similar comparison between temperature anomalies at Kew for the periods 1870-1900 and 1901-1929, so there is a possibility that persistent temperature anomalies may occur over the Continent of Europe without extending to the British Isles.

8. The significant temperature anomalies

It would be premature to attempt a full scale discussion of the temperature singularities listed in Table 4. The network of stations was not chosen to show the significance of singularities and there is no guarantee that this list is complete. Nevertheless, it is worthwhile to consider what has been proved, and some possible directions of future investigation. In the first place, the

/hypothesis

hypothesis that the temperature anomalies are due to local causes independent for each station and period is decisively disproved. On this hypothesis the occurrence of an anomaly of given sign in corresponding periods in each of the 15 records shown in Figs. 5 to 9 should occur only once in 2^{15} investigations, and the number of cases of complete or near complete agreement among the 73 comparisons actually made is far greater than the chance expectation.

Looking now at Fig. 5, no one would have been surprised to find the amount of agreement between the anomalies which actually exists had the stations been spaced, say, within a 50-mile radius. What is interesting is that the anomalies should agree to the extent which they do over an area extending from London to the Baltic. It seems that the processes which produce these anomalies work on a scale rather larger than that of a European country, and approaching the size of the half waves in the general westerly circulation. Further, it appears as if most of the region considered has some kind of climatic homogeneity since when the anomaly in a given period has the same sign at all except one station, the exceptional station is usually one on the northern or eastern borders of the area. If we had to outline the area in which this homogeneity might be expected, we should exclude Glasgow, Edinburgh, Uppsala and Tilsit, but include Kew, Potsdam, Vienna, the whole of Germany, and an area to west of the Rhine not covered by present observations.

9. A particular singularity

The most obvious feature in the cycle is the occurrence of positive anomalies in late May and the first few days of June, followed by negative anomalies in late June. This succession is clearly seen on Fig.5, for example, in the records for Darmstadt and Potsdam, but it occurs in each of the records within the area described above, and is responsible for three of the widespread anomalies listed in Table 4. The magnitude of the temperature anomaly is largest in the German records, where the positive and succeeding negative anomalies may be nearly 1°C. At Kew the anomalies are similar but a good deal smaller, while they are very small at Edinburgh and Glasgow. At Tilsit and Uppsala the anomalies are smaller and the change from positive to negative anomalies is smoother and less abrupt than at the Central European stations.

This singularity is evidently a phenomenon primarily affecting Central Europe, and becoming less important as we go north and east.

In any part of the world, the climate at a given time of year is the result of several weather types, some warmer, and some colder than the average, and to get an average 5-day mean temperature anomaly of 1°C, which is the figure actually attained in Central Europe, must mean a considerable excess of warm weather types in May followed by an excess of cool types in June. Thus the singularity in Central Europe implies a tendency of features of the large-scale circulation to persist in fixed positions during the months of May and June, and if this happens in Central Europe, the weather conditions over the British Isles will be materially affected by it. Hence the weather during these months in the British Isles may be influenced by the processes producing a temperature singularity which is most strongly developed in Central Europe.

In this case it is easy to suggest a physical cause for the singularity, namely, a monsoonal effect arising from the solar heating of the European land mass, but if we are to forecast the character of the phenomenon from year to year, we should probably look at the area where it is most intense, namely, Central Europe, rather than a marginal region such as the British Isles, to find what makes it slight or vigorous, late or early.

10. Discussion

The first part of the investigation shows that over the large area examined the annual temperature variation can be represented for most purposes by two

harmonic terms. This conclusion has been used in a later paper in which normal temperatures at 5-day intervals throughout the year are given for a network of stations covering the entire Northern Hemisphere. The methods used have been programmed for electronic computation, so that the work can be repeated with slight modifications for every weather element we wish to investigate, and we intend to do this as time permits. Further work projected will deal with the variability of weather elements at any given time of year, and this should complete the statistical ground-work for the attack on the problem of weather anomalies.

The second part of the investigation, which suggests that the annual temperature variation shows small persistent departures from the two term harmonic form at particular times of the year, is an unexpected by-product of the first. The question whether these departures will recur in the future cannot be settled beyond doubt from the present evidence, but there is no doubt that during the period covered by the historical record some of the departures occurred at the same time over a very large area, on the same scale of the half waves of the planetary circulation. We cannot, of course, deduce that disturbances of the planetary circulation are prone to occur in particular areas at particular times of the year, but at least the evidence is consistent with the existence of such disturbances.

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Lewis and McIntosh	1952	Met. Mag., London, Vol. 81, p. 242.
Pollak	1948	Harmonic analysis and synthesis schedules, Dublin Meteorological Service.
Quetelet	1867	Memoire de l'Academie Royale Belgique, 36.

Table 1. The stations used

Station	Latitude	Longitude	Altitude m.	Period over which data are averaged	Source of data
Augsberg	48°22'N	10°54'E	502	1851-1930	Deutsche Klimakünde
Bayreuth	49°57'N	11°34'E	364	1851-1930	Deutsche Klimakünde
Berlin	52°31'N	13°22'E	62	1851-1930	Deutsche Klimakünde
Breslau	51°07'N	17°02'E	147	1851-1930	Deutsche Klimakünde
Brussels	50°52'N	04°23'E	39	1831-1860	Quetelet (1867)
Brocken	51°48'N	10°37'E	1150	1896-1930	Deutsche Klimakünde
Darmstadt	49°53'N	08°40'E	150	1851-1930	Deutsche Klimakünde
Dresden	51°04'N	13°46'E	112	1851-1930	Deutsche Klimakünde
Edinburgh	55°57'N	03°15'W	76	(1795-1804 (1821-1850	Forbes (1860)*
Emden	53°22'N	07°12'E	8	1851-1930	Deutsche Klimakünde
Erfurt	50°58'N	11°04'E	218	1851-1930	Deutsche Klimakünde
Frankfurt a.O.	52°20'N	14°35'E	57	1851-1930	Deutsche Klimakünde
Friedrichshafn	49°39'N	09°29'E	408	1851-1930	Deutsche Klimakünde
Glasgow	55°52'N	04°17'W	56	1868-1912	M.O.M. Vol.III
Görlitz	51°10'N	15°00'E	217	1851-1930	Deutsche Klimakünde
Gütersloh	51°55'N	08°23'E	81	1851-1930	Deutsche Klimakünde
Halle	51°29'N	11°59'E	94	1851-1930	Deutsche Klimakünde
Halle	51°29'N	11°59'E	94	1851-1900)	Met. Zeitschrift
Halle	51°29'N	11°59'E	94	1901-1950)	
Hohenpeissenberg	47°48'N	11°01'E	994	(1851-1874 (1879-1930	Deutsche Klimakünde
Kew	51°28'N	00°19'W	8	1871-1950	M.O. Tabulation, 1951. Averaged in M.O.22
Kiel	54°19'N	10°08'E	47	1851-1930	Deutsche Klimakünde
Kleve	51°48'N	06°08'E	48	1851-1930	Deutsche Klimakünde
Köln	50°56'N	06°57'E	56	1851-1930	Deutsche Klimakünde
Königsberg	54°43'N	20°30'E	7	1851-1930	Deutsche Klimakünde
Konitz	53°42'N	17°34'E	175	1851-1930	Deutsche Klimakünde
Köslin	54°12'N	16°11'E	46	1851-1930	Deutsche Klimakünde
Lüneberg	53°15'N	10°24'E	20	1851-1930	Deutsche Klimakünde
Memel	55°43'N	21°08'E	18	1851-1930	Deutsche Klimakünde
München	48°10'N	11°30'E	531	1851-1930	Deutsche Klimakünde
Posen	52°25'N	16°56'E	66	1851-1930	Deutsche Klimakünde
Potsdam	52°23'N	13°04'E	80	1901-1950	Met. Zeitschrift
Prague	50°05'N	14°25'E	197	1775-1839	Hlavac (1937) worked in M.O.22
Prague	50°05'N	14°25'E	197	1840-1934	Hlavac (1937)
Ratibor	50°06'N	18°13'E	191	1851-1930	Deutsche Klimakünde
Schneekoppe	50°44'N	15°44'E	1618	1881-1930	Deutsche Klimakünde
Stettin	53°26'N	14°30'E	26	1851-1930	Deutsche Klimakünde
Stuttgart	48°47'N	09°10'E	267	1851-1930	Deutsche Klimakünde
Tilsit	55°04'N	21°54'E	18	1851-1930	Deutsche Klimakünde
Torgau	51°34'N	13°00'E	99	1851-1930	Deutsche Klimakünde
Treuberg	54°02'N	22°30'E	162	1851-1930	Deutsche Klimakünde
Trier	49°45'N	06°38'E	150	1851-1930	Deutsche Klimakünde
Uppsala	59°51'N	17°38'E	24	1871-1950	Year books, worked in M.O.22
Vienna	48°15'N	16°22'E	203	1873-1947	" "
Zugspitze	47°25'N	10°59'E	2962	1901-1930	Deutsche Klimakünde

* Observations made at three locations less than 1¾ mi. apart. Elevation from 70-79 m.

Table 1a. The Norwegian stations used (after Birkeland (1936))

Station	Latitude	Longitude	Altitude m.	Source of data
As	59°40'N	10°46'E	95	Geofysiske Publikasjoner Vol. XIV No. 1
Bodo I.	67°17'N	14°24'E	22	"
Dalen I.	59°27'N	07°58'E	103	"
Dombas	62°05'N	09°07'E	643	"
Hamar	60°48'N	11°04'E	139	"
Hattfjelldal	65°36'N	14°00'E	235	"
Kristiansand N.	63°07'N	07°45'E	18	"
Kristiansand S.	58°10'N	07°59'E	22	"
Nordfjordeid	61°55'N	06°00'E	5	"
Oslo	59°55'N	10°43'E	22	"
Skudenes	59°09'N	05°16'E	2	"
Sorvaranger	69°44'N	30°03'E	5	"
Tromso I.	69°39'N	18°58'E	45	"
Trondheim	63°26'N	10°25'E	58	"
Ullensvang	60°19'N	06°40'E	10	"
Vollen	61°06'N	08°58'E	403	"

Table 2. Harmonic coefficients, total and residual variances

Station	Annual mean temperature and harmonic coefficients in degrees Celsius a_0	a_1	b_1	a_2	b_2	Annual variance	Residual variance	Residual variance as percentage of total	No. of years
Augsberg	8.12	-9.54	-2.59	-0.33	0.45	49.21	0.16	0.33	80
Bayreuth	7.76	-9.42	-2.55	-0.22	0.28	47.81	0.12	0.25	80
Berlin	9.27	-9.35	-2.73	0.11	0.28	47.59	0.14	0.29	80
Breslau	8.58	-9.88	-2.94	-0.06	0.16	53.25	0.13	0.24	80
Brussels	10.28	-7.87	-2.66	-0.02	0.33	34.80	0.23	0.66	30
Brocken*	2.63	-6.83	-2.92	0.41	0.26	27.86	0.16	0.57	35
Darmstadt	9.61	-8.76	-2.22	-0.05	0.36	40.93	0.06	0.15	80
Dresden	9.14	-9.04	-2.71	0.00	0.29	44.71	0.18	0.40	80
Edinburgh	8.31	-5.68	-2.14	0.21	0.48	18.65	0.08	0.43	40
Emden	8.52	-7.65	-2.85	0.15	0.43	33.51	0.12	0.36	80
Erfurt	7.93	-8.94	-2.61	-0.07	0.24	43.58	0.19	0.44	80
Frankfurt a.O.	8.54	-9.50	-2.77	0.07	0.24	49.19	0.17	0.35	80
Friedrichshafn	8.84	-9.21	-2.65	-0.13	0.26	46.08	0.12	0.26	80
Glasgow	8.29	-5.10	-2.04	0.45	0.54	15.36	0.05	0.33	45
Görlitz	8.24	-9.43	-2.83	-0.41	0.21	48.70	0.18	0.37	80
Gütersloh	9.05	-8.11	-2.49	0.16	0.29	36.22	0.17	0.47	80
Halle (1851-1930)	9.07	-9.22	-2.63	0.07	0.30	46.12	0.15	0.33	80
Hohenpeissenberg*	6.00	-8.11	-2.91	0.12	0.43	37.49	0.23	0.61	76
Kew (1901-1950)	10.29	-6.23	-2.39	0.34	0.62	22.63	0.10	0.44	50
Kiel	8.47	-7.72	-3.04	0.43	0.36	34.69	0.10	0.29	80
Kleve	9.13	-7.75	-2.45	0.11	0.41	33.26	0.14	0.42	80
Köln	10.24	-8.00	-2.44	-0.02	0.38	35.19	0.15	0.43	80
Königsberg	7.06	-9.90	-3.47	0.22	0.16	55.20	0.15	0.27	80
Konitz	6.81	-9.85	-3.11	0.27	0.15	53.53	0.16	0.30	80
Köslin	7.22	-8.82	-3.21	0.26	0.30	44.21	0.14	0.32	80
Lüneberg	8.47	-8.51	-2.61	0.31	0.27	39.89	0.17	0.43	80
Memel	6.85	-9.51	-3.89	0.31	0.14	52.97	0.07	0.13	80
München	7.64	-9.53	-2.65	-0.36	0.36	49.18	0.12	0.24	80
Posen	8.29	-9.95	-2.89	0.17	0.12	53.86	0.16	0.30	80
Potsdam	8.54	-9.35	-2.65	-0.08	0.12	47.44	0.15	0.32	50
Prague (1775-1839)	9.57	-10.33	-3.01	-0.35	0.08	58.16	0.18	0.31	65
Prague (1840-1934)	9.04	-10.00	-2.83	-0.13	0.20	54.12	0.09	0.17	95
Ratibor	8.05	-9.97	-2.80	-0.22	-0.01	53.69	0.09	0.17	80
Schneekoppe*	0.24	-7.28	-3.07	0.36	0.19	31.60	0.28	0.89	50
Stettin	8.41	-9.37	-2.95	0.19	0.29	48.48	0.14	0.29	80
Stuttgart	10.00	-8.98	-2.35	-0.16	0.41	43.34	0.13	0.30	80
Tilsit	6.59	-10.53	-3.40	0.32	-0.05	61.41	0.14	0.23	80
Torgau	9.01	-9.35	-2.72	0.08	0.29	47.62	0.18	0.38	80
Treuberg	5.43	-10.83	-3.23	0.30	-0.37	64.22	0.21	0.33	80
Trier	9.17	-8.22	-2.27	-0.06	0.40	36.58	0.12	0.33	80
[illegible]	5.25	-9.96	-3.63	0.97	0.25	56.73	0.05	0.09	80
[illegible]	9.38	-10.09	-2.77	-0.50	0.19	54.89	0.06	0.11	75
[illegible]ze*	-4.92	-6.10	-3.19	0.22	0.16	24.12	0.39	1.62	30

* denotes mountain stations.

Table 2a. Harmonic coefficients, total and residual variances for Norwegian stations (after Birkeland (1936))

Station	Annual mean temperature and harmonic coefficients in degrees Celsius a_0	a_1	b_1	a_2	b_2	Annual variance	Residual variance*	Residual variance as percentage of total*	No. of years
As	4.96	-10.10	-3.24	0.65	0.18	56.44	0.04	0.07	60
Bodo I.	4.11	-6.85	-3.24	0.86	0.78	24.49	0.09	0.31	60
Dalen I.	5.07	-9.90	-2.98	0.75	0.32	53.75	0.06	0.11	40
Dombas	1.04	-10.23	-3.06	0.56	0.50	57.28	0.07	0.12	60
Hamar	3.44	-11.19	-3.56	0.48	-0.03	69.07	0.04	0.06	52
Hattfjelldal	0.91	-10.84	-3.63	0.53	0.62	65.83	0.17	0.26	52
Kristiansand N.	6.59	-5.39	-2.74	0.60	0.58	18.63	0.03	0.16	60
Kristiansand S.	6.88	-8.11	-3.12	0.83	0.25	38.09	0.04	0.11	44
Nordfjordeid	6.03	-7.25	-2.27	1.14	0.48	29.82	0.17	0.57	15
Oslo	5.84	-10.58	-3.18	0.68	0.17	61.21	0.02	0.03	60
Skudenes	7.51	-5.55	-3.21	0.45	0.40	20.77	0.04	0.19	60
Sorvaranger	-0.71	-10.92	-4.44	0.36	0.68	70.01	0.21	0.30	60
Tromso I.	2.40	-6.79	-3.15	1.09	0.90	29.02	0.05	0.17	60
Trondheim	4.78	-8.14	-2.86	0.63	0.58	37.57	0.06	0.16	51
Ullensvang	6.68	-7.50	-2.56	0.88	0.30	31.83	0.04	0.13	60
Vollen	1.89	-11.44	-3.56	0.08	-0.01	71.81	0.17	0.24	60

* This value for the residual variance is half the sum of the squares of Birkeland's values for the semi-amplitudes of harmonics 3, 4, 5 and 6, and therefore is less than the true value.

Fig 1(a) M.R.P. 915

Multipliers for evaluating Harmonic Co-efficients from 73 equally spaced values of a function

Term No.	$\frac{2}{73}\cos\theta$	$\frac{2}{73}\cos 2\theta$	$\frac{2}{73}\cos 3\theta$	$\frac{2}{73}\cos 4\theta$		$\frac{2}{73}\sin\theta$	$\frac{2}{73}\sin 2\theta$	$\frac{2}{73}\sin 3\theta$	$\frac{2}{73}\sin 4\theta$
1	274	273	272	270		012	024	035	047
2	272	265	254	238		035	070	103	135
3	268	249	219	179		059	114	165	208
4	262	226	170	098		081	155	215	256
5	254	196	109	006		103	192	251	274
6	244	160	041	-087		125	222	271	260
7	232	120	-029	-170		145	246	272	215
8	219	076	-098	-232		165	263	256	145
9	204	029	-160	-268		183	272	222	059
10	187	018	-212	-272		200	273	174	-035
11	170	-064	-249	-244		215	266	114	-125
12	150	-109	-270	-187		229	251	047	-200
13	130	-150	-273	-109		241	229	-024	-251
14	109	-187	-258	-018		251	200	-092	-273
15	087	-219	-226	076		260	165	-155	-263
16	064	-244	-179	160		266	125	-208	-222
17	041	-262	-120	226		271	081	-246	-155
18	018	-272	-053	265		273	035	-269	-070
19	-006	-274	018	273		274	-012	-273	024
20	-029	-268	087	249		272	-059	-260	114
21	-053	-254	150	196		269	-103	-229	192
22	-076	-232	204	120		263	-145	-183	246
23	-098	-204	244	029		256	-183	-125	272
24	-120	-170	268	-064		246	-215	-059	266
25	-140	-130	274	-150		235	-241	012	229
26	-160	-087	262	-219		222	-260	081	165
27	-179	-041	232	-262		208	-271	145	081
28	-196	006	187	-274		192	-274	200	-012
29	-212	053	130	-254		174	-269	241	-103
30	-226	098	064	-204		155	-256	266	-183
31	-238	140	-006	-130		135	-235	274	-241
32	-249	179	-076	-041		114	-208	263	-271
33	-258	212	-140	053		092	-174	235	-269
34	-265	238	-196	140		070	-135	192	-235
35	-270	258	-238	212		047	-092	135	-174
36	-273	270	-265	258		024	-047	070	-092
37	-274	274	-274	274					

Fig. 1(b) Form for Harmonic Analysis of function given by 73 equally spaced values

Term No.				sum	diff.
1	73				
2	72				
3	71				
4	70				
5	69				
6	68				
7	67				
8	66				
9	65				
10	64				
11	63				
12	62				
sums					
sums of squares					
13	61				
14	60				
15	59				
16	58				
17	57				
18	56				
19	55				
20	54				
21	53				
22	52				
23	51				
24	50				
sums					
sums of squares					
25	49				
26	48				
27	47				
28	46				
29	45				
30	44				
31	43				
32	42				
33	41				
34	40				
35	39				
36	38				
sums					
sums of squares					
37					

Mean & Variance

	Sum	Sum of squares
Block 1		
Block 2		
Block 3		
Term 37		
Total		

Mean =

Variance =

=

=

Cosine component

	$\frac{1}{10} a_1$	$\frac{1}{10} a_2$
Block 1		
Block 2		
Block 3		
Term 37		
Total		

a_1 =

a_2 =

Sine components

	$\frac{1}{10} b_1$	$\frac{1}{10} b_2$
Block 1		
Block 2		
Block 3		
Total		

b_1 =

b_2 =

Amplitudes & phases

$A_1 = \sqrt{a_1^2 + b_1^2} =$

$\phi_1 = \tan^{-1} b_1/a_1$

$A_2 = \sqrt{a_2^2 + b_2^2} =$

$\phi_2 = \tan^{-1} b_2/a_2$

Residual variances

$V_1 = V_0 - \frac{1}{2}(a_1^2 + b_1^2) =$

$V_2 = V_1 - \frac{1}{2}(a_2^2 + b_2^2) =$

$\frac{V_2}{V_0} =$

FIG 2(ω) SEMI-AMPLITUDE OF FIRST HARMONIC, IN DEGREES CELSIUS.

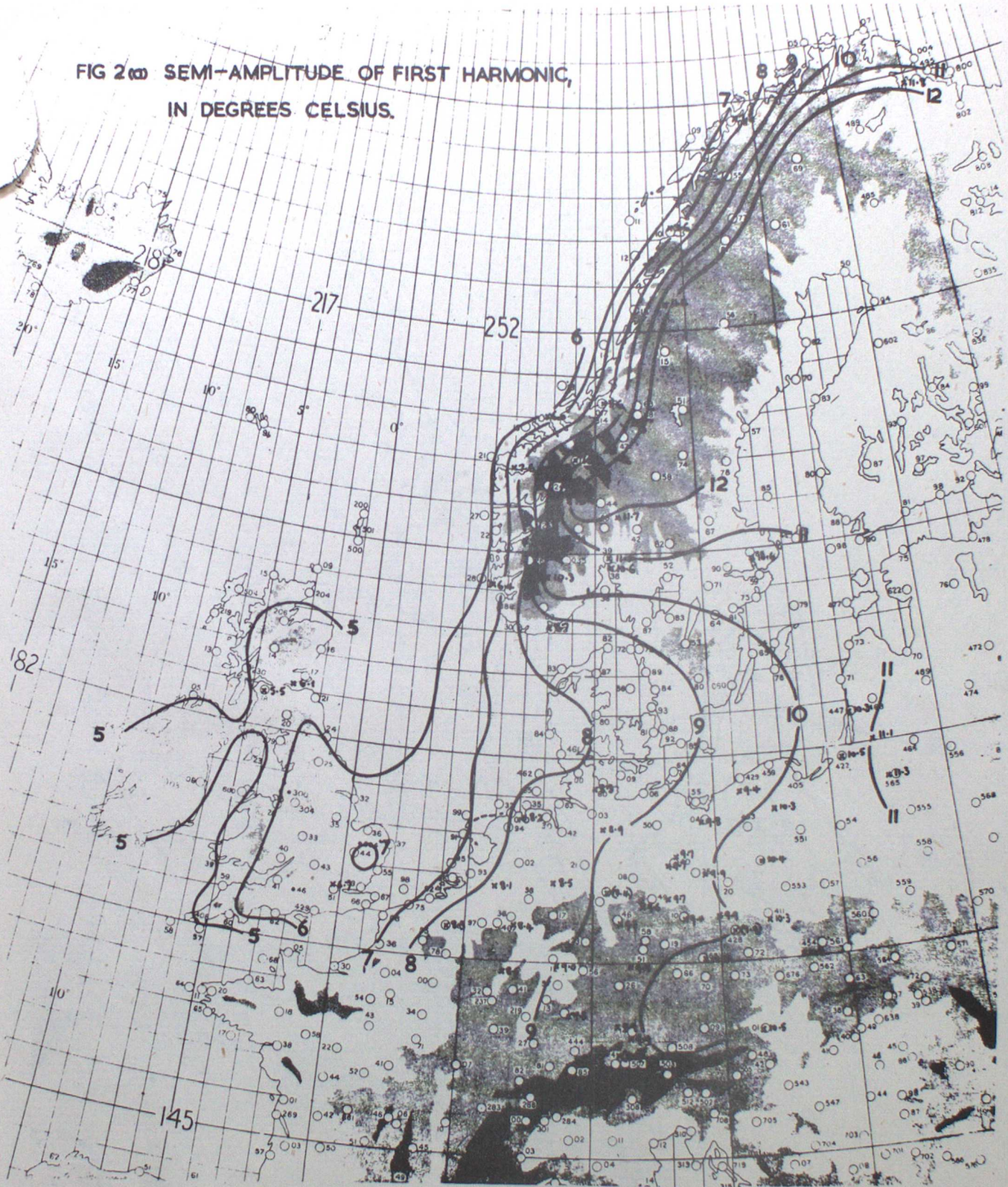


FIG 2 (b) PHASE OF FIRST HARMONIC,
DATE OF MAXIMUM.

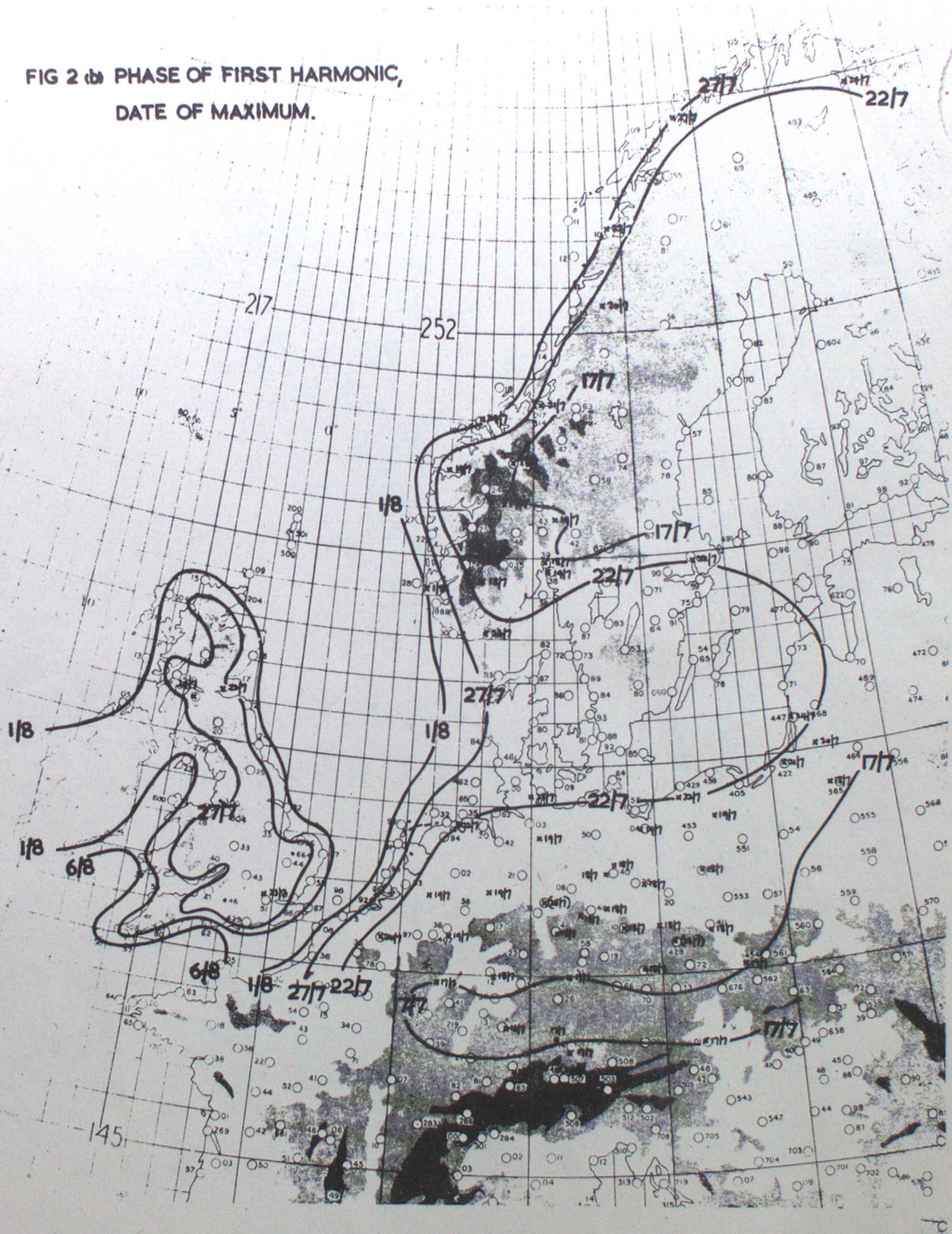


FIG 2 — SEMI-AMPLITUDE OF SECOND HARMONIC, IN DEGREES CELSIUS

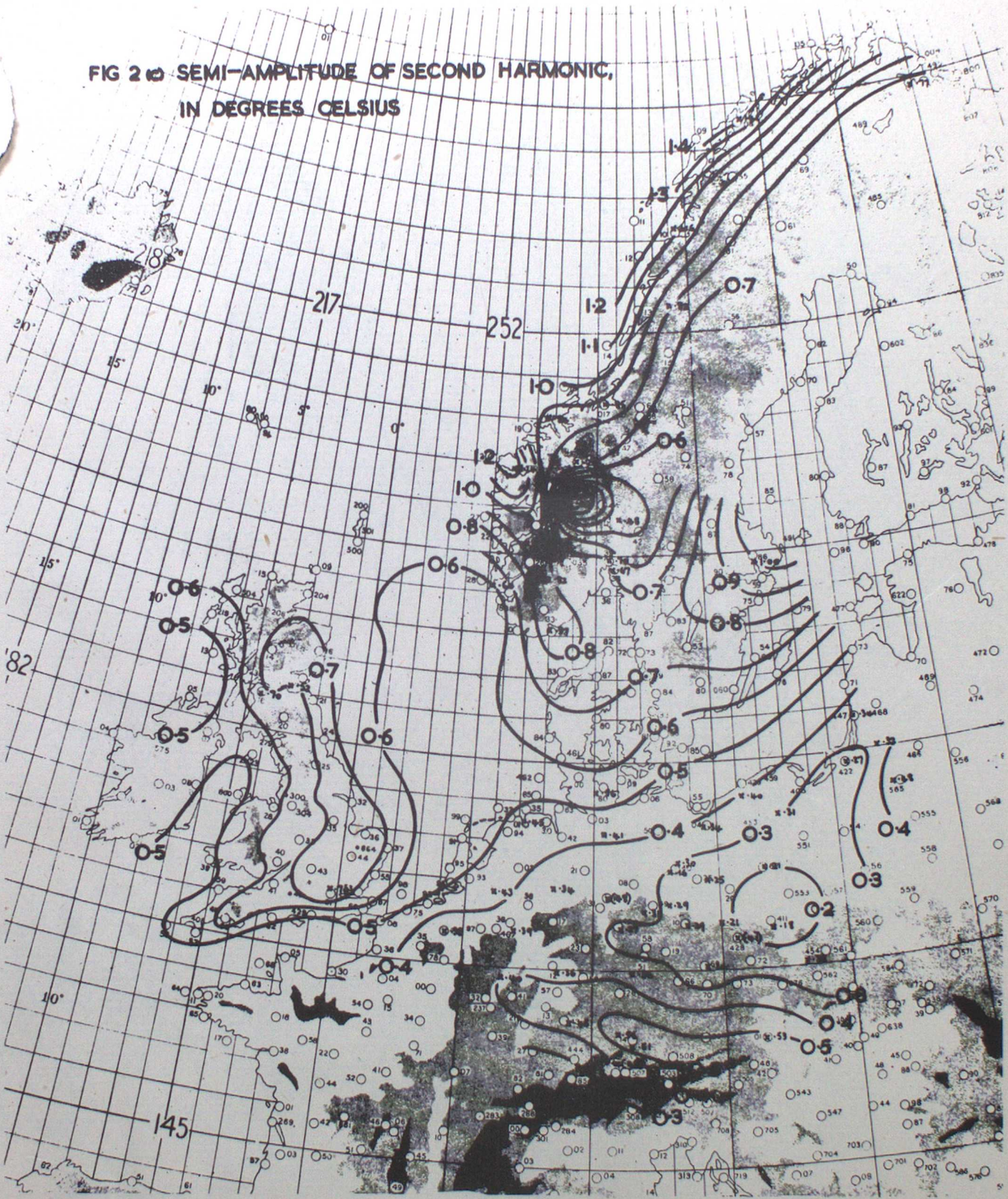


FIG 2(d) PHASE OF SECOND HARMONIC, DATE OF MAXIMUM.

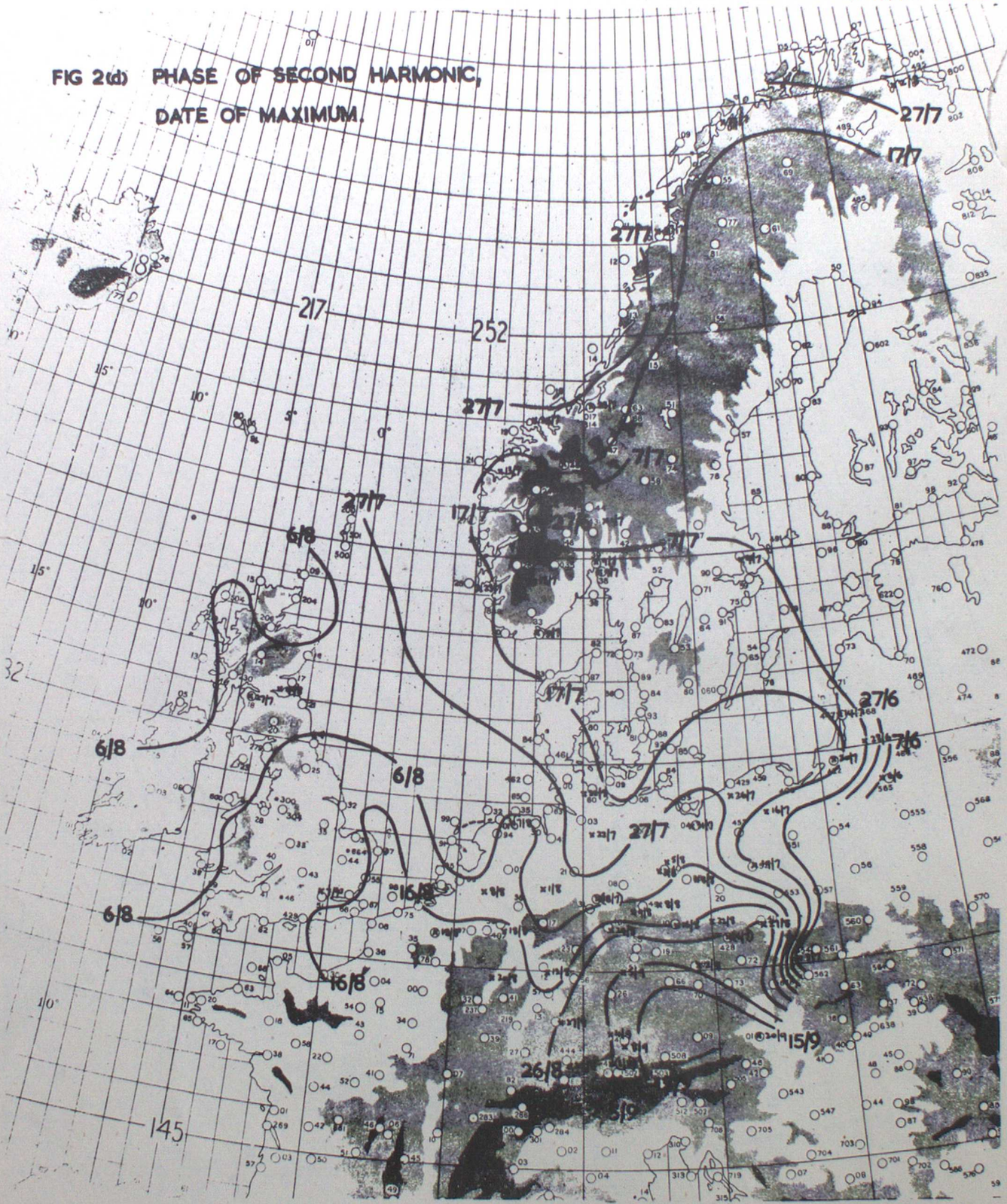


FIG 2(a) POSITION, ALTITUDE & ANNUAL MEAN TEMPERATURE OF STATIONS.

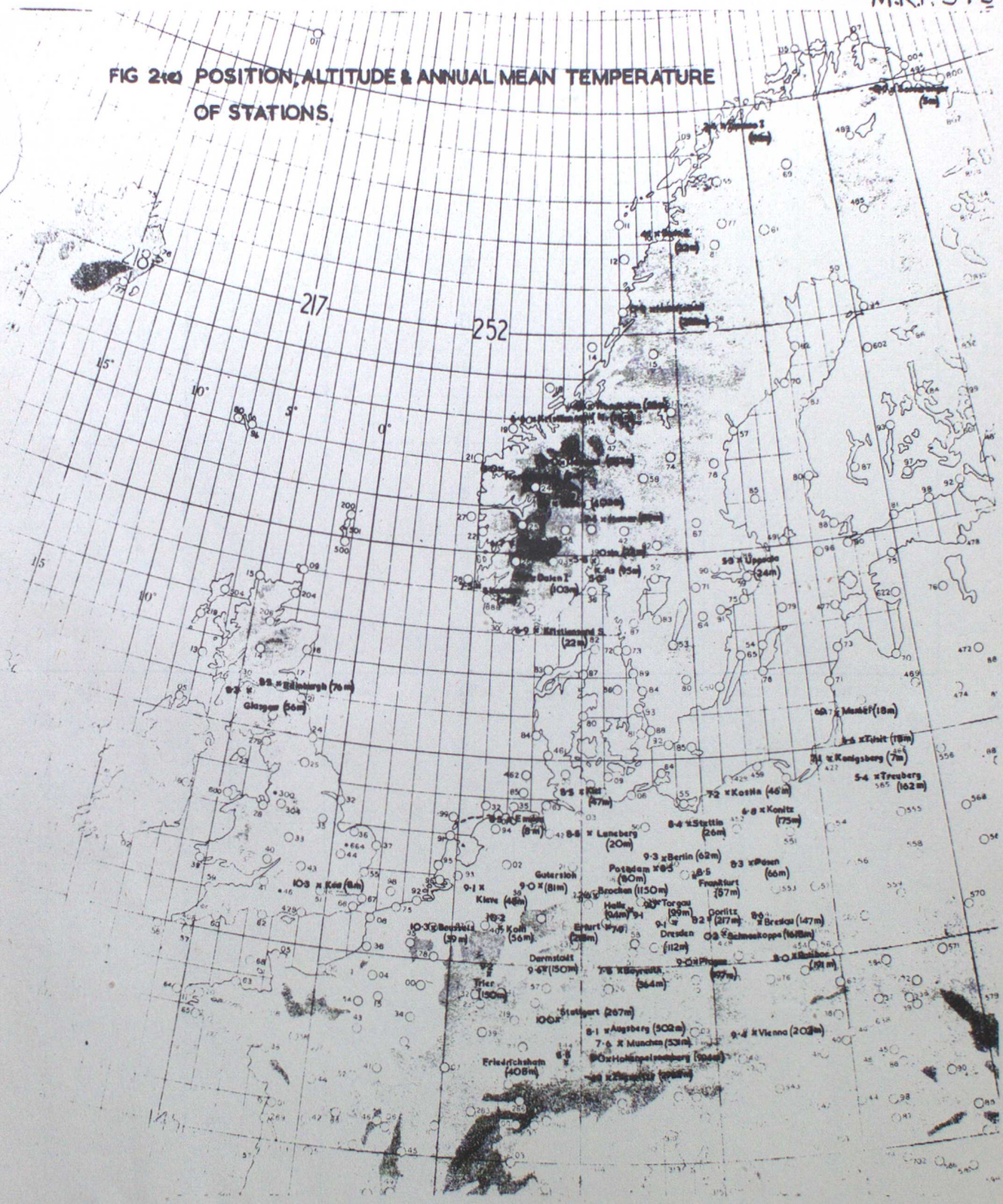


Fig. 3 The relation between nV and n.

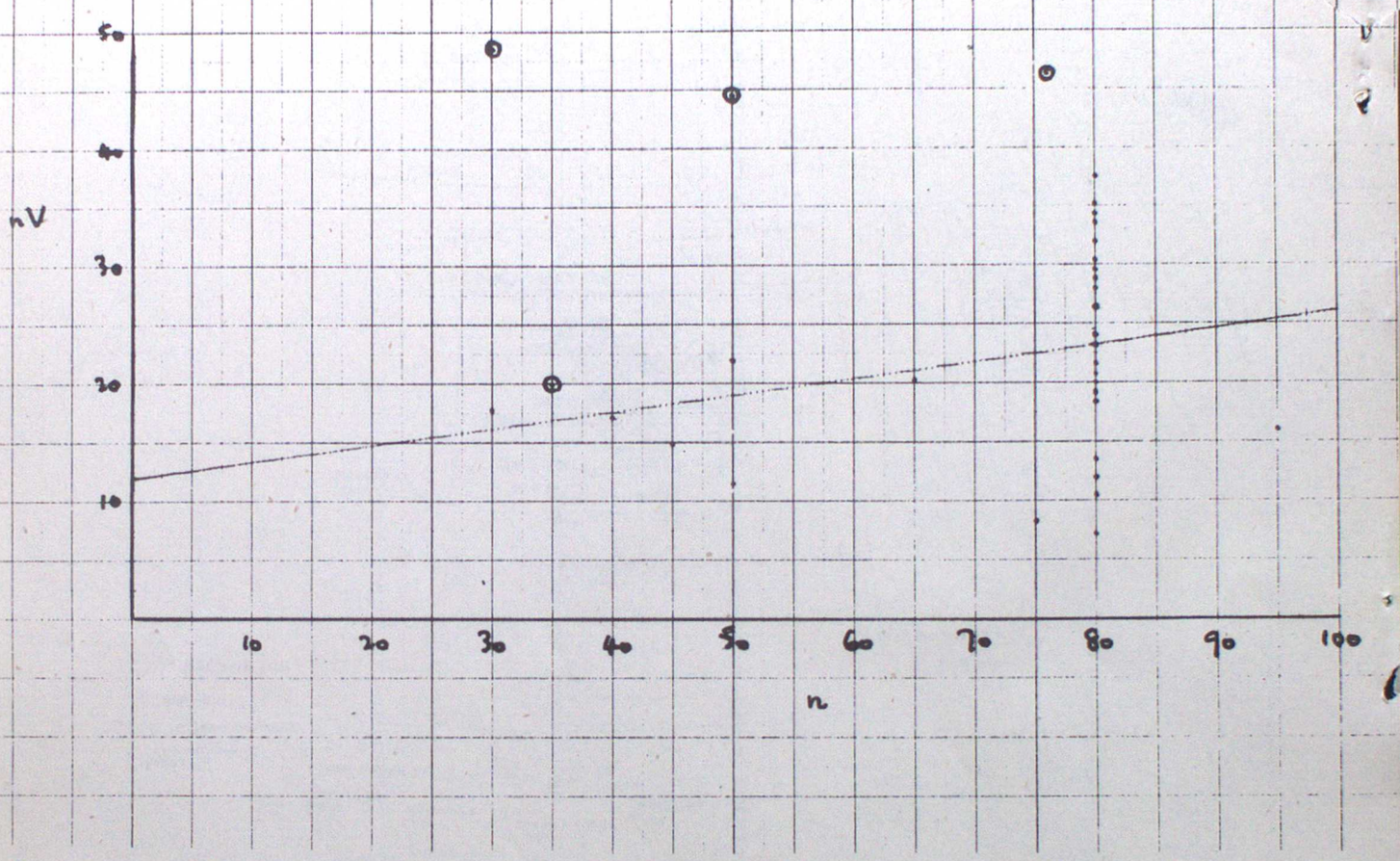


FIG.4 GEOGRAPHICAL DISTRIBUTION OF PERCENTAGE RESIDUAL VARIANCE x NUMBER OF YEARS.

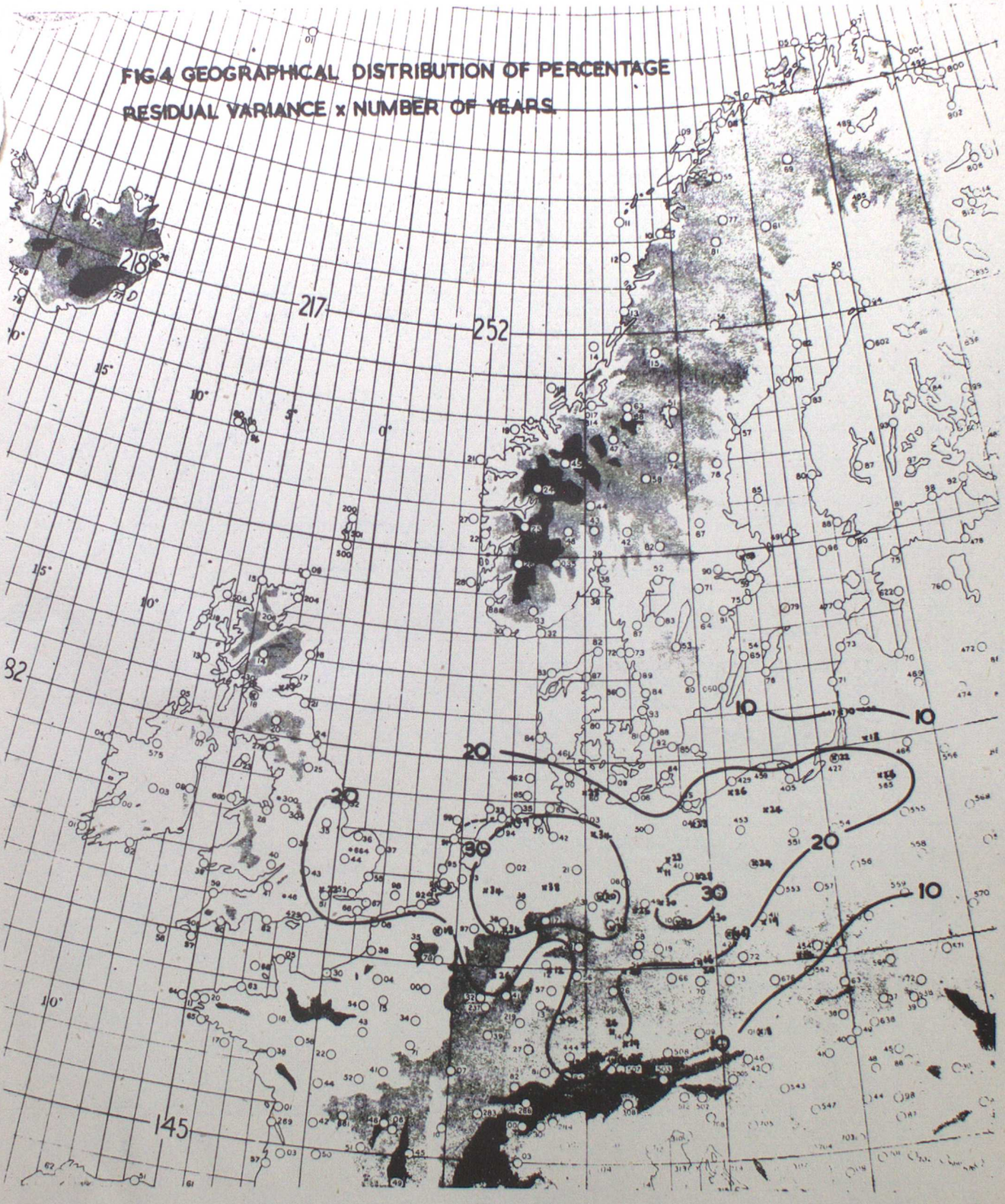


FIG 5

ANOMALIES OF 5 DAY MEAN TEMPERATURES FOR 8 STATIONS IN NORTH-WEST & CENTRAL EUROPE.

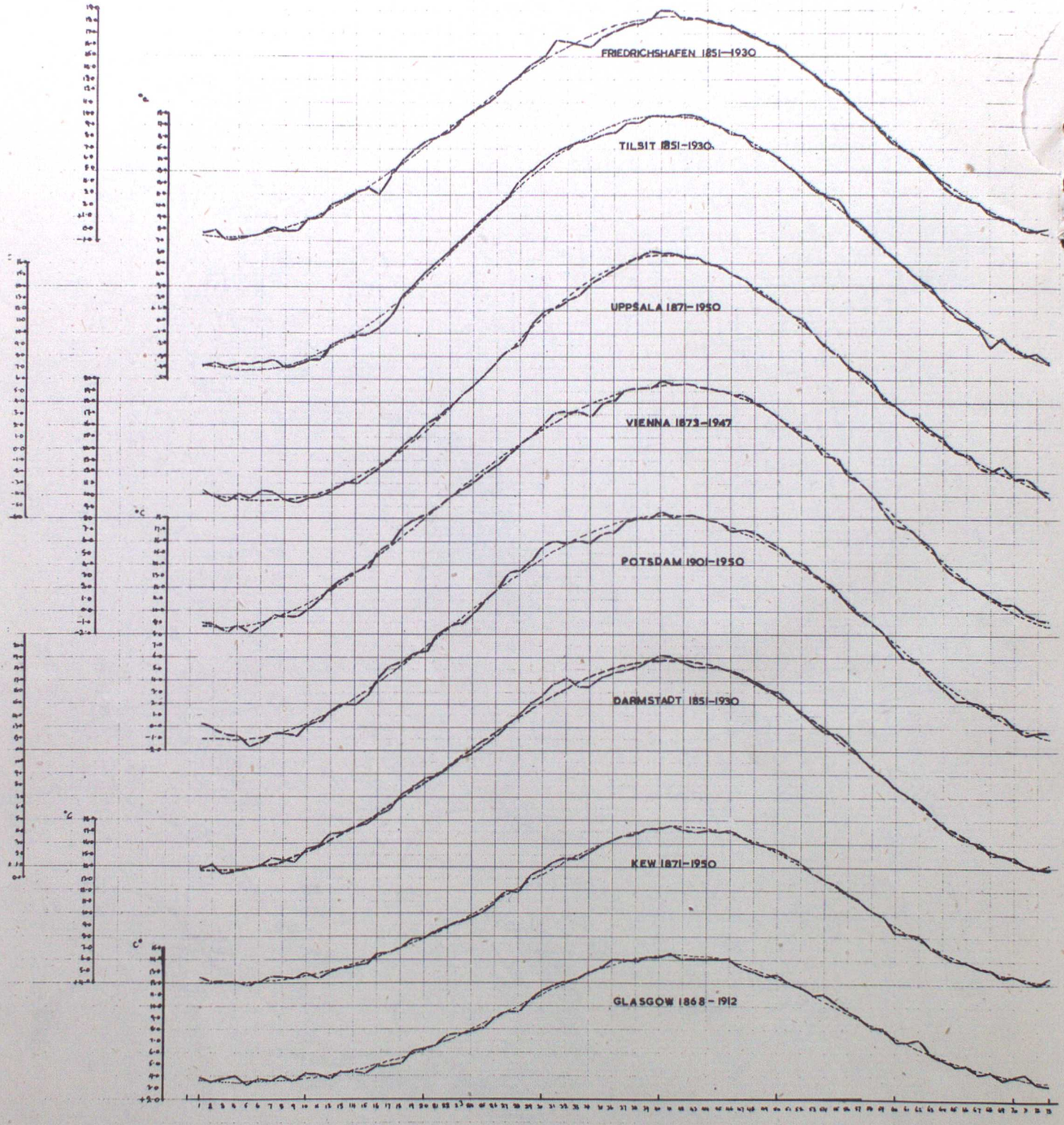


FIG 6

TEMPERATURE ANOMALIES AT PRAGUE (50°05'N, 14°25'E, 197·2 M. A.S.L.) 5DAY MEAN TEMPERATURES COMPARED WITH THE TWO TERM HARMONIC NORMALS FOR THE PERIODS 1775–1889 & 1840–1934.

NORMAL FOR 1775–1839 $(9{\cdot}54 - 10{\cdot}33\cos\Theta - 3{\cdot}01\sin\Theta - 0{\cdot}35\cos 2\Theta + 0{\cdot}08\sin 2\Theta)$ °C
NORMAL FOR 1840–1934 $(9{\cdot}04 - 10{\cdot}00\cos\Theta - 2{\cdot}83\sin\Theta - 0{\cdot}13\cos 2\Theta + 0{\cdot}20\sin 2\Theta)$ °C

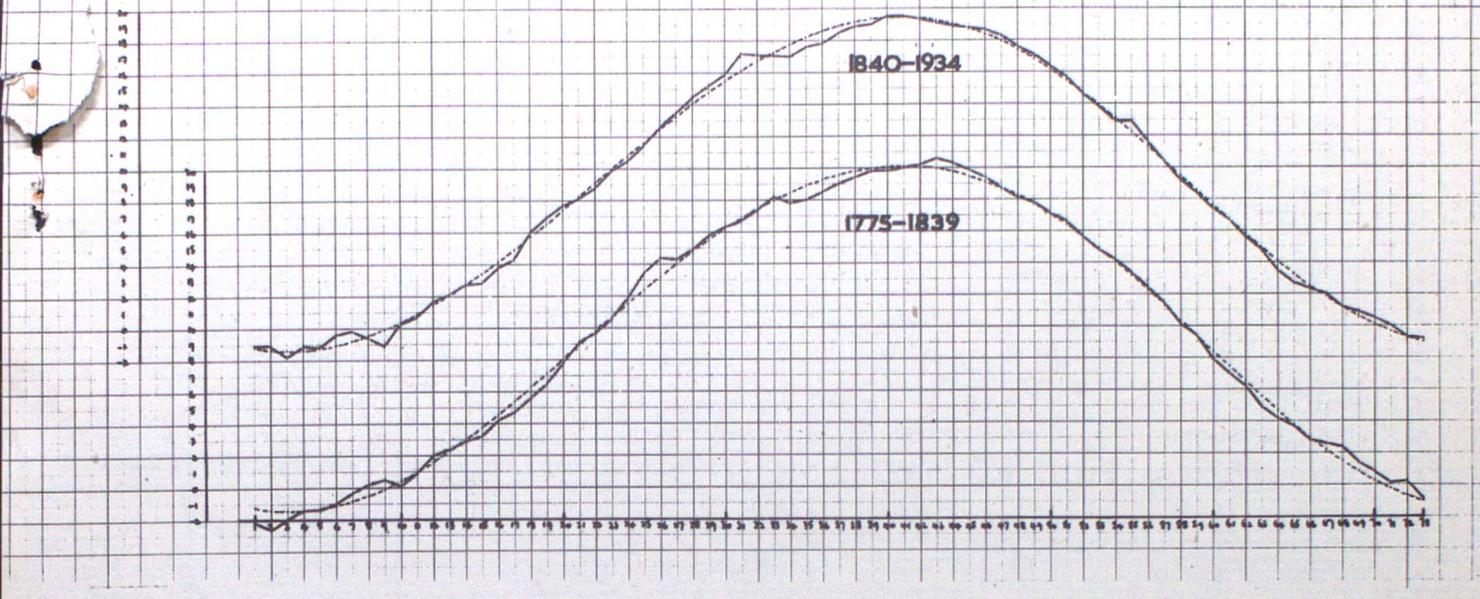


FIG 7

TEMPERATURE ANOMALIES AT HALLE S. (52°29'N 11°58'E. 90·8 M. A.S.L.) 50 YEAR 5 DAY MEAN TEMPERATURES COMPARED WITH THE TWO TERM HARMONIC NORMALS FOR THE PERIODS 1851–1900 & 1901–50.

NORMAL FOR 1851–1900 $(8{\cdot}95 - 9{\cdot}44\cos\Theta - 2{\cdot}79\sin\Theta + 0{\cdot}04\cos 2\Theta + 0{\cdot}36\sin 2\Theta)$ °C
NORMAL FOR 1901–50 $(9{\cdot}42 - 9{\cdot}13\cos\Theta - 2{\cdot}57\sin\Theta - 0{\cdot}05\cos 2\Theta + 0{\cdot}15\sin 2\Theta)$ °C

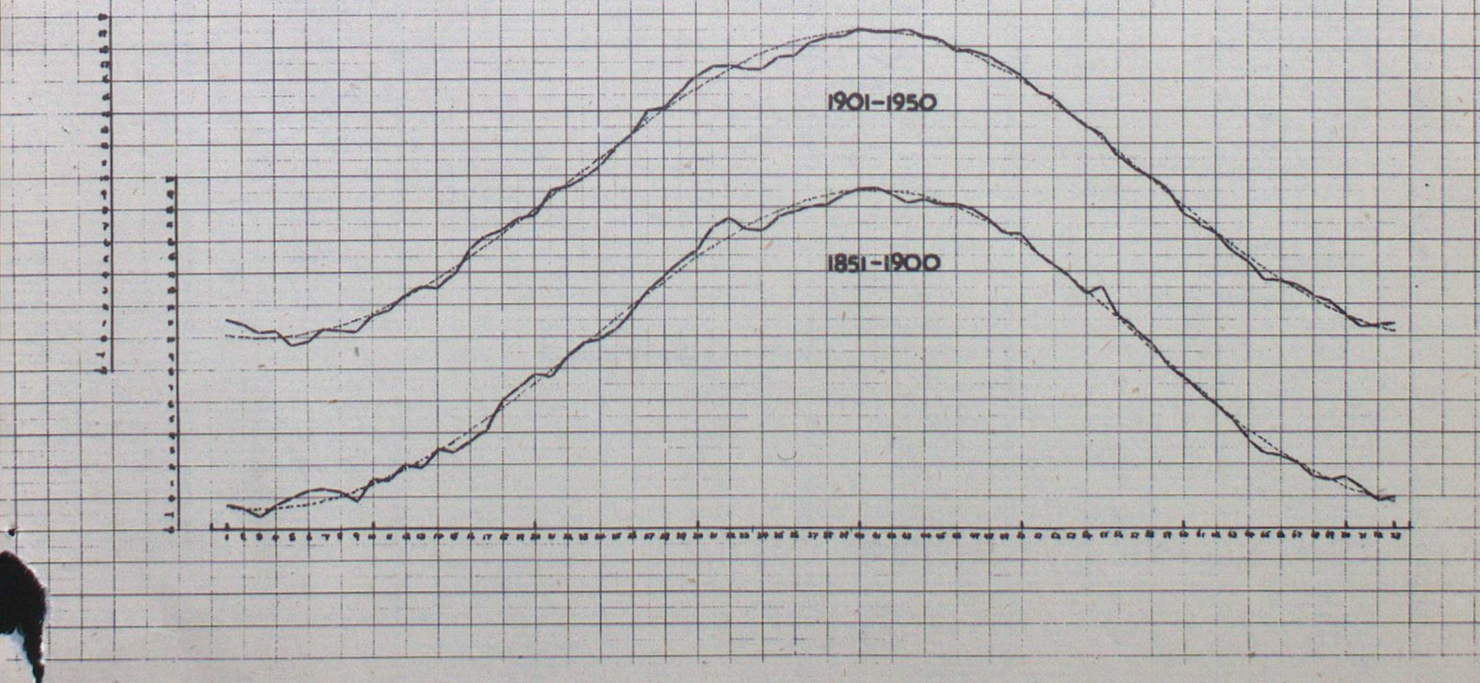


FIG 8

5 DAY MEAN TEMPERATURE ANOMALIES FOR EDINBURGH (55°57′N, 3°11′W) 1795–1804, 1821–1850 COMPARED WITH GLASGOW (55°51′N, 4°16′W) 1868–1912.

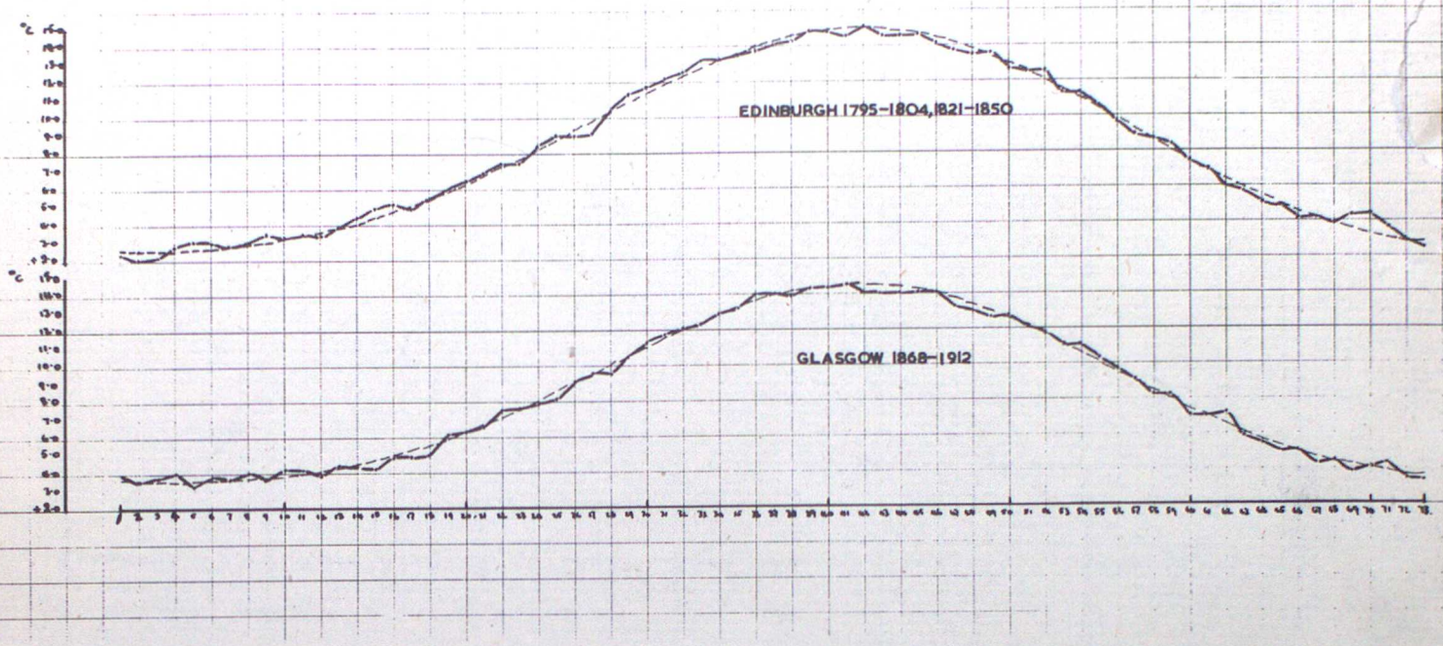
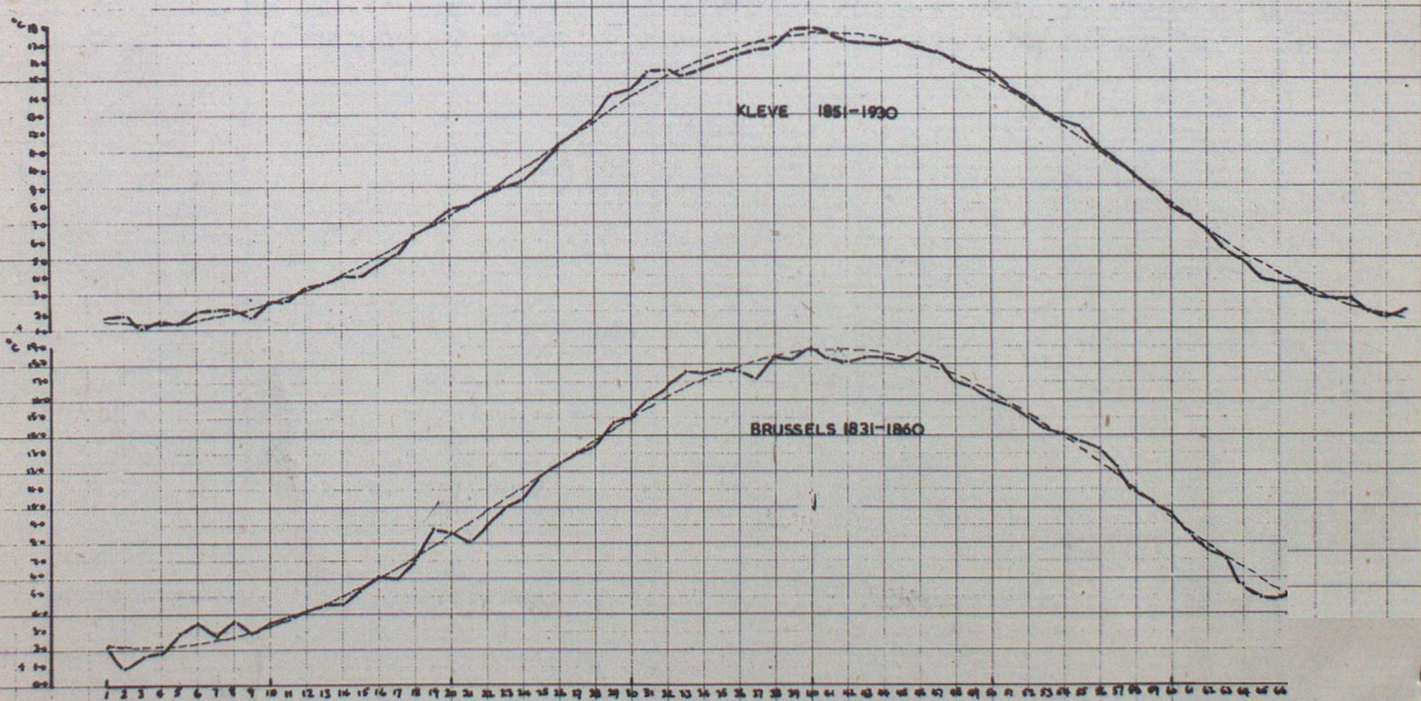


FIG 9.

5 DAY MEAN TEMPERATURE ANOMALIES FOR KLEVE (51°48′N, 6°8′E) 1851–1930 COMPARED WITH BRUSSELS (50°51′N, 4°22′E) 1831–1860.